Lecture 1

Introduction to Proofs

Prep:

* Write bigger, fewer lines on board
* Check test date(s) and number of problem sets
* Court cases
* Course video taping
* Colbert video—test ahead
* Handouts (course info, Globe article)
* Lecturer names
* PS 1 info (drop-off location, time, format, boxes)
* Course URL & Website (with sign up area for recitations)
* Get there 30 minutes early!!—test video

Take

* Handouts (course info, Globe article)
* DVD and laptop with Colbert video (pre-test)
* Check clock

**6.042/18.062 “Proofs, Proofs, & more Proofs”**

**Lecturers: Tom Leighton & Ankur Moitra \*\***

**Handouts:**

* **Course Info**
* **Globe article**

**Notices:**

* **Course website:** [**https://courses.csail.mit.edu/6.042/fall16/**](https://courses.csail.mit.edu/6.042/fall16/) **\***
* **Register preferences for recitation assignment on website by 7:30pm tonight.**
* **PS #1 (on website tonight) is due 7:30 pm Monday 9/12!! \*\***
* **Read Ch. 1 & Sec 2.0 – 2.4 of text (link on website)**
* **Recitation tomorrow (sections on website by 10pm tonight!)**

**Write grading scheme on board**

**Check writing size in back of room**

Welcome to 6.042/18.062 Discrete Math for CS (more affectionately known as Proofs, Proofs & more Proofs!). It probably won’t take you long to figure out how the course got this nickname. **☺**

Can you hear me in the back?

My name is Tom Leighton and I will be co-teaching this term with **Prof Ankur Moitra. \*\***

**Stand up.**

Info on how to contact us is on class website. **Point to URL.** My office hours are TR 4:30-5:30 in 32G – 594 on days when I lecture. I’ll be out of town on most days when I don’t lecture, so check the course calendar on the web site to make sure I will be there before you come.

Everything you will need to know is on the course web site. There is also some hi-level information on the course info sheet that we handed out at the start of class. In fact let’s go through info on that handout now.

**Show it.**

BTW: Everyone have the 2 handouts?

Pre Req. for this course is 18.01

If you have had 18.200, 18.310 or 6.046, you are probably too advanced for this class and should not be here – see us after class if you have questions.

This is a 5-0-7 course so you’ll be spending lots of time in class.

Lecture T R 2:30 – 4:00 in this room. We’ll start at 2:35 but it’s a big class so try to be here @ 2:30 to get handouts & get settled.

Attendance: at lecture is not required but it is highly recommended. The text covers most everything but it does not include all the questions & discussions that come up during lecture – so it’s a good idea to attend lecture and take notes. Over the years, we’ve found that students who come to lecture have an easier time absorbing the material and they do better as a result.

Recitations: Wed & Fri. Attend 1 each day. Will be small (15 or so students) so that you can have closer interaction with staff.

1st part of recitation will usually cover material related to previous day’s lecture. Most of recitation will be focused on problem solving – split into groups of 3 – 4 students and solve problems as a team. Great chance to develop problem solving skills and to learn from classmates. Hopefully it will even be fun from time to time. Also a good way to meet other students to work on homework with later.

There is good news and bad news with recitations.

Bad News: Attendance is required and accounts for 20% of your grade.

Good News: The good news is you get a perfect score just for showing up and participating. **Explain point system:** 1 point for showing up. 2 points for showing up and actually saying something. Now, if you show up late and then proceed to snore loudly, then you are probably not going to get 2 points. ☺

Recitations are the closest thing to a gimmie that you’ll get in this class so be sure to go! (can’t emphasize enough).

We will schedule you into a recitation tonight based on the preferences you give us on-line. Be sure to sign up by 7:30pm tonight!

Recitation assignments will be posted on Web site tonight … by 10pm if all goes well.

First recitation is tomorrow! If you can’t attend your assigned recitation tomorrow or if all does **not** go well and we can’t get the schedule done in time, then go to another section and tell a TA so we can reschedule you.

Questions?

There is 1 text for course – Mathematics for Computer Science. It’s available for free on course website \*\*. It is a work in progress so please let us know if you find any bugs. We’ll assign reading from the text along with each problem set. Lots of reading at start of term, In fact, you should read most of the first 2 chapters this week. It will probably be review for many of you.

2 Exams- One 2-hour midterm scheduled instead of lecture on October 25th, and one 3-hour final during finals week.\*\*

Exams will be fairly challenging and will be curved, but only upwards. If median < 75%, we’ll normalize to make it 75%. Never curve downward.

12 Problem sets: **\*\***

Usually posted on website after class on Tuesday and due at 7:30 pm the following Monday night **(PS 1 is due next Monday).** It’s pretty short—just to get you going and up to speed on basic definitions and notation.

Turn them in in 5th Floor elevator lobby of Gates 32G **\*\*.**  There will be a separate box for each problem. This means that you need to write your name and your TA’s name on each problem and you can’t staple them all together.

We’ll try to have them graded and returned by Friday recitation. We’ll post solutions on Tuesday, which means that you can’t turn in late! If you have a good reason why you can’t get homework done on time, see your recitation instructor in advance and they may be able to help.

Be neat and clear in homework. Graders \*\* will be reading hundreds of solutions each week and they start marking off points for sloppiness pretty early on. (They’re students too and, like you, they have a million things to do … 200th problem late at night & too sloppy to read – you can imagine what they’ll do….) **☺**

You are allowed to collaborate on homework – in fact, we encourage it!. But you must understand and write up solutions yourself. You must also cite your collaborators on your homework. No penalty for collaboration – as long as you tell us and you write up your own solution in your own words.

Should not copy old homework and you should not use bibles. But if you do use another written source, then acknowledge it by saying so on the problem set. You shouldn’t do it, but we won’t penalize you--we’d rather that you learn it somehow than not at all.

You can and should ask your TA’s and the staff for help – they will all have office hours every week. You can go to anyone’s office hours if the hours for your recitation instructor don’t work out. The TAs will help you get to the right answer. You don’t need to cite TA’s as collaborator. The more you can do on your own or by working with other students, the better, but we will give you all the help you need and you should feel free to ask for it.

Tell story of students camped out in my office for HW—doesn’t scale well.

Study groups are a great way to learn material and do well in course. Useful to catch errors in homework and help solve harder problems. And some of the problems **are** hard. We don’t expect you to solve every homework problem all by yourself, but we do want you to work through every problem and to write up a correct solution. No reason you can’t get perfect score on every homework. HW meant as a learning aide – not as an aide in grading you.

In fact, our whole philosophy in teaching this course is that we are here to help you learn, not because we like grading you. Unfortunately, the department requires that we assign grades and so we need to do that.

Along these lines, I should probably be clear that collaboration is **not** allowed on exams, **☺** so don’t do it. If you do collaborate on an exam, it is much better that you tell us before we find out some other way! We know pressure can be intense and we can be understanding – so don’t cheat. But if you do, tell us.

Questions?

Problems on tests will be similar to homework problems & recitation problems – should not require knowing a trick like sometimes happens in homework. Doing well on homework is great way to do well on test.

Grades for the course are calculated by a fairly complicated algorithm – described on website

**Grading\*\***

**20% Tutorial 88 – 100 A**

**50%**

**30% Problem Sets 75 - 88 B**

**25% Midterm 60 - 75 C**

**35% Final 50% 50 - 60 D**

**<50 F**

**Q:** Anyone notice anything wrong with this?

**A:** 110% **-** we’ll knock off 10% of your worst exam score – very helpful if you mess up on one test. So your grade will be half from Exams & half from tutorial and coursework.

Write 50-50 on board

Also, we’ll **knock out** lowest homework score and 2 lowest recitation scores.

So don’t panic if you get over loaded at some point during the term (you can blow off one week and be ok). But if it starts happening on a regular basis, come see us.

The key to doing well in this course it to attend lecture & recitation and work hard on homework. No reason everyone can’t get an “A” – we won’t curve downward.

Now it probably won’t turn out that everyone will get an “A”, but when we have used this grading policy in the past, there have been a large number of “A’s”. Actually, department sometimes not too happy about it. In fact, one time they asked me what the heck I thought I was doing giving so many “A’s” **☺**, but on other hand students really did work hard and did really well on exams … So it turned out OK.

Questions?

The Content of the course is roughly divided into four (4) parts:

**First Part** of course covers proof techniques. As you might have gathered from the Underground Guide, there is a heavy emphasis on proofs in course. Our most important goal in this course is to teach you how to recognize a fallacious argument and how to formulate a correct proof.

**Second Part** of course covers discrete structures for modeling computing systems. Heavy emphasis on graph theory & networks. Also a heavy emphasis on proofs.

**Third Part** of course focuses on tools for counting & estimation. Less emphasis on proofs and more on methods of analysis. This material will be very useful later when you need to estimate the running time of an algorithm in 6.006 or 6.046 or the complexity of solving a difficult computation problem in later life.

**Last Part** of course focuses on discrete probability. The material we are going to cover in probability is related to the material in other probability courses at MIT but it is more focused on the kinds of discrete problems that come up in computer science. So instead of spending time on topics like the normal distribution, we’ll cover topics like chernoff bounds.

When we are done with the course we would like you to be able to do 6 things:

1. Understand what a **proof** is and how to do one
2. Understand basic **discrete structures** (like networks) and how to reason about them
3. Understand how to **abstract a problem** and then solve it
4. Understand **probability** and be able to work with it
5. Develop a good **bag of math tricks** for analyzing problems that come up in computer science
6. Develop a **good sense** for **spotting flaws** in arguments.

You know, in school, you get used to believing what you are told. Usually this is ok, because for the most part, we try to teach you things that are true. ☺ But believing what you are told can be a bad habit to get into, especially once you get out of school and into the business world, because in real life, people are going to tell you untrue things all the time. And understanding when something is true … or not … can be really important.

The best way to learn how to spot flaws is by studying examples, so we’re going to show you a lot of false arguments during the term. In most cases, the argument will have been in the press or even a famous academic paper and it is going to sound great, but it will fall apart under closer inspection. Our goal is to get you used to questioning what you are told and to validating whether or not it is really true.

Now usually, when we show you a bogus argument in class, it will have been planned … but not always **☺** .

Tell story of the card trick a few years ago.

Basically the content of this course can be boiled down into good news and bad news.

Bad News: is that this is a math course. Even worse, 6.042 is a *required* math course. **☺**

Good News: is that we realize that most of you are not math majors and that you are not here because of your undying love for abstract math **☺**. So for the most part, we’ll be covering material that will be useful to you in later computer science courses. Now at times its going to seem like the material we are covering is hopelessly abstract and completely useless, but believe it or not, that view will only be partially correct **☺**. A lot of the stuff we’re going to do really is useful… Really!

We’ll also try to have some fun from time to time in class. Now, of course, there is only just so much fun that you can have in a math class. To be honest, I’ve always been jealous of my colleagues who teach intro physics and chemistry. In chemistry, you can mix chemicals & make liquids change color & even have small explosions. In physics, you can wheel out a Van de Graph generator and wire up a student and make their hair stand on end. But there is a limit to what we can get away with in Math. There’s just no chance of seeing one of your classmates get electrocuted in this class, it simply won’t happen **☺** – but we’ll try to keep it from being too boring.

The class is going to be fairly fast paced. There is a lot of material that the Dept. wants you to know and only 14 weeks to get it all covered so you’ll need to keep up.

Many of you will already be familiar with material we are covering at the beginning. That’s OK, but be sure not to fall into trap of falling asleep for too many lectures – since we’ll be moving on to new and less familiar material very soon. Vital to keep up – otherwise you can get into trouble very quickly. Oddly enough, this problem tends to hit the better students—they already know all about truth tables and induction so they fall asleep at the beginning and by the time they wake up a few weeks later, they’ve missed number theory & graph theory and they’re doomed.

Questions?

OK, let’s start talking about proofs. Everyone has seen pfs before, even way back in 9th grade geometry. At its core,

**A proof is a method of ascertaining the truth.**

By ascertaining the truth, I mean establishing truth or verifying truth. Now there are lots of ways to ascertain the truth in society and even within science.

Q. Can anyone think of some? \*\*

**Ex:**

**Legal** Judge or jury. (How many people have heard of OJ? His trial was one of the most watched of all time and featured a special math defense counsel—we’ll talk more about it later in the term.) \*\* In any case, OJ Simpson was “not guilty” of murdering his wife Nicole– although a lot of people think he did it, but using the “legal” method of ascertaining the truth, he was found to be not guilty.\*\*

**Religious** (word of God) very hard to argue about because you just believe it. And unless you have a direct line to the Almighty, you have to rely on others to interpret for you – like minister, priest, rabbi or ayatollah **☺** And, of course, they can have conflicting views and so you can end up with conflicting truths, which makes it very complicated.

**Business** (what the boss says) If Donald Trump is your boss, you better agree or “your fired”. In business, the customer is always right. What they say becomes “truth.”

Related Ex: Professor says so! Not in this class. One of the nice things about math is that the youngest student can win an argument with most senior professor. In fact, I’m always very happy when a student can prove me wrong – well maybe not always. **☺** Can be a little embarrassing but I usually end up learning something as a result—as with the card trick example I mentioned before.

**Experiment & observation** (physics) Who really knows if gravity exists, but we believe it to be true because we can observe it and measure it.

**Sampling** (statistics) If it happens 10 times in a row or if you can’t find a counterexample, then it must be true. Bedrock principle of many of the sciences.

Inner **Conviction** – very popular – especially in computer science – “there aren’t any bugs in my program”. **☺**

Closely related is: “I don’t see why not” Always a favorite when a student tries to justify why their answer is right. Shifts burden of proof to anyone who disagrees with proposition.

Ok, in math we strive for a higher threshold before we accept something as being true. Formally: **A mathematical proof is a verification of a proposition by a chain of logistical deductions from a base set of axioms.**

**SAVE**

Bit of a mouthful. Three 3 key components to a mathematical proof:

Propositions, axioms, and logical deductions

We’ll spend the rest of class talking about each component, starting with propositions.

**Def: A proposition is a statement that is either true or false.**

**Ex: 2 + 3 = 5** This is a true proposition

Here is a bit more complicated example:

**Ex: ∀n∈N : n2+n+41 is a prime number**

{not div by any other # besides itself & 1}

Ex: 2,3,5,7,11,13 are prime;

but 4 is not a prime



**“For All” {0, 1, 2, ….}**

**“Natural “Numbers”**

This part of proposition is called a **Predicate: proposition**

**whose truth depends on value of variables.** When you have “for all” or “exists”, then there is a predicate.

“Universe of discourse” for qualifier.

How many have seen this notation?

Point

To see if this proposition is true, we need to see if the predicate is true for every choice of n. So let’s check:

**n n2+n+41 prime?**

**0 41 T**

**1 43 T**

**2 47 T**

**3 53 T**

**:**

**20 461 T**

**:**

**39 1601 T**

Ok must be true, right? Works for 1st 40 Examples!

What do you think? Raise your hand if you think it is true. Raise your hand if you think it is false.

1. Can anyone find an n for which n2+n+41 is not prime?

**A.** Easy **412+41+41 = 41 ∙ 43 F**

Also 402+40+41 = 1681 = 412

This is a great example: Very common in some fields to conclude something is true by just checking a bunch of examples. – Physics, engineering, CS: simulations used in lieu of proof.

Here, the first 40 examples worked – but that’s not enough.

There are some examples in number theory where 1st counterexample is even worse.

**Ex:** **a4+b4+c4 = d4 has no positive integer solution.**

Euler conjectured this in 1769

Big honcho in math. Still talk about him a lot, even though dead for centuries.

For over 200 years, no one knew if prop was true or false. Conjecture was ultimately disproved a few years ago by a fellow named Noam Elkies (rather clever fellow that works at that liberal arts school down the street). It turns out that the equation is satisfied by:

**a = 95800**

**b = 217519**

**c = 414560**

**d = 422481**

Or so Noam says ….. **☺**

Don’t worry—you don’t have to remember these numbers—we won’t quiz you on stuff like that.

So a true proposition is:

**∃ a,b,c,d,∈N +, a4+b4+c4=d4 .**

**“there exists”** Predicate—truth depends on values of a,b,c,d

Positive integers

Here’s another example from number theory:

**Ex: 313 (x3+y3) = z3 has no positive integer solution.**

Turns out that this prop is False: but smallest counter example has over **1000** digits!

No hope to resolve this problem by computer search--you could try random values of x and y for zillions of years and still not find the counter example.

Of course, some of you may be thinking – why on earth would I care if there is positive integer solution or not! Who could possibly care about this except for a few weird mathematicians? That will probably not be the last time the thought occurs to you this term **☺.** And who cares about **1000-digit** numbers anyway?—they are way too large to think about.

But actually, in this case, the solution to problem was important! This equation is an example of an elliptic curve, which is central to understanding how to factor large 1000-digit integers. And if you can factor 1000-digit integers, you can break some of the most widely-used cryptosystems like RSA. And if you can do that, you can hack into most every computer system on the planet.

So, more important than you might first think. In fact, that will often be the case this term: at first it might seem pretty abstract, but it will turn out to be useful later on in future courses or your career.

We’ll talk more about factoring and cryptosystems in a couple of weeks when we study number theory.

OK – back to propositions … Here is a famous one:

**Ex. The regions in any map can be colored with 4 colors so that adjacent regions have different colors.**

Known as 4-color thm.

1. How many of you have heard of it?

Long history: first conjectured by Guthrie in 1853.

Many false proofs over time. One of the most convincing was a proof using pictures by Kempe in 1879. It was believed for over a decade until Heawood found a flaw in the argument. Proof consisted of drawing pictures of what maps had to look like and then Kempe argued from there. Proofs by picture are often very convincing but are also often very wrong. We’re going to give a very simple but very wrong proof by picture on Tuesday.

4-color thm was finally “proved” in 1977 by Appel & Haken with aide of computer to check lots of cases – but some mathematicians don’t believe it because it’s hard to check a proof by computer – how do you know code worked? Still no bugs yet found & so seems ok. Somewhat simpler proofs were discovered later but all of them still require a computer to check some cases. Most mathematicians now agree that the 4-color theorem has been proved, but a human-checkable proof would be nicer.

Here is another famous one:

**Ex: Every even number but 2 is the sum of two primes.**

**Ex: 24 = 11 + 13**

**Q:** T or F?

**A:** No one knows. True for all number ever checked and they’ve checked a zillion by computer. It’s generally thought to be true but don’t really know for sure – all it takes is one bad even number to disprove conjecture.

Known as Goldbach’s Conjecture after Christian Goldbach who first stated proposition in 1742. Very simple proposition, and because of its simplicity, lots of people have tried to figure out if it’s true. In fact, I spent two years working on it many years ago—I thought it had to be easy to solve because it was so easy to state. I’m mean, how hard can it really be?

Turns out that it seems pretty hard—Listed as one of great unsolved mysteries on the front page of Globe in mid 90’s.

1. See handout – Anyone notice anything strange about Globe write-up on Goldbach’s Conjecture?

20 = 9 + 11. 9 is not exactly a prime **☺** … Can’t believe what you in read in the paper. We’ll see even worse examples later during the term.

Article also lists two other famous conjectures, which most people believed to be true.

**Read Rieman \*\***

(1.5 trillion zeros not enough to prove it!)

Was claimed to be proved in 2008, but proof turned out to be bogus.

**Read Poincare**

Conjecture roughly says that 3-d objects without holes are equivalent to the sphere – Known to be true for 4 dims. & higher. Turns out Poincore’s conjecture was proved to be true in 2003 by Russian named Grigor Perelman. Some controversy around Perelman, however, since his 80-page proof did not have all the details. Then other teams of mathematicians came up with 350 page proofs, claiming to have all the details. Most experts now agree than Perelman’s proof is right & so he was recently named the first winner of a $1 million Millennium prize.

Perelman also won Fields Medal – very prestigious prize, awarded to top mathematicians every 4 years at ICM. Perlman won it at the 2006 ICM but he declined prize which created a big controversy in mathematics. And then he also declined the $1M Millennium Prize!

Anyway, the whole affair is rather murky and so to make it a little clearer I have brought a video from an expert on the subject.

**SHOW VIDEO**

OK, back to propositions. Have you all heard the expression “if pigs could fly, then I would be king”?

Q. Is this a true proposition?

A. Almost—we just need to put it in a more mathematical form. In particular, if/then statements can generally be recast as implications.

**Def: An implication has the form p ⇒ q where p and q are**

“IMPLIES”

**propositions.**

Ex: **“pigs fly” ⇒** “**I am king”** is a way to represent the old saying as an implication.

An implication is defined to be true if the first part is F or if the second part is T.

**Def: p ⇒ q is true if p is F or if q is T.**

**Truth table for p ⇒ q**

|  |  |  |
| --- | --- | --- |
| **p** | **q** | **p ⇒ q** |
| **T** | **T** | **T**  SAVE |
| **T** | **F** | **F** |
| **F** | **T** | **T** |
| **F** | **F** | **T** |

Now in this case, the proposition **“pigs fly”** is **F** and so the implication **“pigs fly” ⇒** “**I am king”** **is True** whether or not I am king!

Show F rows in table

In fact,

“False **⇒** anything” is True

which seems a little strange, but that is how implications are defined.

Questions?

In general, if you want to prove an implication **p ⇒ q** is true, you might start by checking if p is false. Show in table. If p is false, then you are done since the implication is true.

Alternatively, you can assume that p is true, and then check if q is true. Show in table. Basically, you just need to be sure that you don’t have a situation where p is T and q is F. Show in table. Sometimes, knowing or assuming p is true might help you show q is true.

Questions?

OK, let’s do one more example with implications:

**∀n∈Z, n ≥ 2 ⇒ n2 ≥ 4**

**Integers = {0, 1, -1, 2, -2, …}**

In order to decide if this proposition is true, we need to check if the implication “**n ≥ 2 ⇒ n2 ≥ 4**” is true for every integer value of n. In other words, we need to check that for every n, either

**n ≥ 2** is false or **n2 ≥ 4** is true.

If **n ≥ 2** is false, we are done for that value of n and it doesn’t matter whether or not **n2 ≥ 4** is true. So we only need to check whether **n2 ≥ 4** when **n ≥ 2**, so to prove the result, we just cut to the chase and consider only integer values of n for which **n ≥ 2**. And, of course,

If **n ≥ 2**, then **n2 ≥ 4** for any integer n.

(Since we can square both sides.)

So this proposition is true.

Questions?

1. Is every sentence a proposition?
2. No! Not every sentence is a proposition.
3. Example?
4. “Hello”. Also, a question does not quality as a proposition since it can’t be assigned a value of T or F.

Questions?

Ok, second component of a proof is the axioms upon which the proof is based. Axioms are examples of propositions—the difference is that we assume they are true.

**Def: An axiom is a proposition that is “assumed” to be true.**

There is no proof that an axiom is true – you just assume it is true because you believe it is a reasonable assumption. In fact, the word **axiom** comes from Greek & means **“to think worthy”,** *not* “**to be true**”.

Now sometimes people say, don’t make assumptions when you are doing math, but this is not right. It is **ok** to make assumptions. In fact, it is unavoidable because you have to start from somewhere. The key in math is to state your assumptions up front for everyone to see.

There are all sorts of axioms used in math.

**Examples:**

* + **If a = b and b = c then a = c** (there is no proof for this but it seems pretty safe, right)

**Leave Space on Board**

* + **Euclidean geometry** has an axiom: **Given a line *l* and a point *p* not on *l***, **there is exactly one line through p parallel to *l*** (known as “parallel postulate”).

Sometimes axioms from one domain contradict axioms from another domain. For example:

* + **Spherical geometry** has an axiom contradicting Euclid’s: **Given a line *l* and a point *p* not on *l*, there is no line through *p* parallel to *l.***
  + So does **Hyperbolic geometry: Given a line *l* and a point *p* not on *l*, there are infinitely many lines through p parallel to *l*.**

How can this be?

Does this mean that some of these fields are bogus?

Nope—it’s OK. None of these axioms is “better” than any other: in fact, all yield equally good proofs.

Of course, different axioms lead to very different theorems. But anyone who agrees with your axioms has to accept the theorems that are derived from them. It’s just that lines are different animals in the worlds of Euclidean and spherical geometry.

Mathematicians have long argued over which axioms are “right” and, of course, it is not clear how to resolve this issue. However, early on mathematicians agreed that any reasonable set of axioms should have two key properties:

**Axioms “should” be**

* 1. **Consistent**
  2. **Complete**

A set of axioms is defined to be consistent if no proposition can be proved to be both true and false.

Clearly a desirable property. Don’t want to spend years proving something True only to have it proved False next day! **☺**

Proofs are meaningless if axioms are not consistent.

A set of axioms is said to be complete if it can be used to prove or disprove every proposition. Want to assume enough in the way of axioms to be able to resolve any question if you work hard enough.

Now you would think that it shouldn’t be too hard to get a set of axioms that satisfy these 2 basic properties – but not so. In fact 2 famous logicians, Russel & Whitehead spent their entire careers trying to find a set of axioms that are consistent and complete for basic arithmetic and couldn’t figure it out. And then something completely unexpected happened. This guy named Kurt Godel came along in 1930’s and proved that it is **not possible** for a set of axioms for arithmetic to be both consistent and complete. This discovery shocked the field – Russell and Whitehead were a bit depressed too. Poor guys – imagine spending your whole life in search of the holy grail and then Kurt shows up and proves that it does not exist. Pretty bad day.

But an Amazing result: if you want consistency (and this is a must) then there will be true facts that can not be proved! (We won’t prove it here – covered in logic courses).

For example, maybe Goldbach’s conjecture is true – but there is no proof!

Now, we’ll try not to assign any of these problems for homework **☺** – even though it may seem like we have sometimes … you may be thinking: hey maybe this is one of those problems with no proof … **no** …

Really is amazing fact: Remember when your parents told you: “if you work hard enough, you can do anything?” Well, not exactly ….

Questions?

Remember to enter your recitation preferences on the web site by 7:30pm!

Get back DVD from A/V guy